**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

*μ* = 45 minutes , *σ* = 8 minutes

*10* minutes are already in the time period of 1 hour from drop off

*Thus* the service time has to be less than (60-10) = 50 minutes

*Let*  X be the servicing time

*If X* is greater than 50 then the commitment is not met

To find P( X > 50)

P(X > 50) = 1 – P(X <= 50) = 1-pnorm(50, mean = 45, sd = 8, lower.tail = TRUE ) (…Rcode)

P(X > 50) = 0.266

the probability that the service manager cannot meet his commitment = 0.266

1. 0.3875
2. 0.2676
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

*μ* = 38 , *σ* = 6 minutes

*44* is 1 standard deviation away from the mean 38

By Central limit theorem, 34% = (68/2) employees will be between 38 and 44

34% of 400 = 136 employees

Also using R-code

1-pnorm(44, mean = 38, sd = 6, lower.tail = TRUE ) we get the probability of employees being above 44

Which is 0.1587

Thus number of employees above 44 is 0.1587\*400 = 63.48

136 > 63.48

Hence, the statement is FALSE

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Probability of employees being under age of 30 = pnorm(30, mean = 38, sd = 6, lower.tail = TRUE )

= 0.0912

Number of employees under age 30 = 0.0912\*400 = 36.48 = 36

Statement is TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Distribution of 2X1 is 2X1 ~ N(2μ, 4σ2 ) and X1+X2 ~ N(2μ, 4σ2)

Thus distribution for both is Normal with same parameters

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

To find P(a < X < b) = 0.99

0.99 = P(a < X < b)

= P(X < b) – P(X < a)

= P(X < b) – P(X > b) Since a and b are symmetric about the mean,

= P(X < b) – 1 + P(X < b)

= 2 \* P(X < b) – 1

P(X < b) = 1.99/2 = 0.995

b = qnorm(0.995, 100, 20)

b = 151.52 = 151.5

Hence, a = 100 – (151.52-100) = 48.48 = 48.5

Answer :

D. 48.5, 151.5

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
8. Specify the 5th percentile of profit (in Rupees) for the company
9. Which of the two divisions has a larger probability of making a loss in a given year?

Let X be the total profit

Then X ~ N(5+7=12, 4\*(32+42)=102)

A. Central 95% of X between X1 and X2

Then, P(X < X2) = 0.95+0.025 = 0.975

X2 = qnorm(0.975, 12, 10)

X2 = $31.6

X1 = 12 – (31.6-12) = $-7.6

Converting to Rupees $1 = Rs. 45

X1 = 45\*(-7.6) = Rs. -342 Million

X2 = 45\*(31.6) = Rs. 1422 Million

Rupee range is Rs. -342 Million to Rs. 1422 Million

B. 5th percentile is given by

X = qnorm(0.05, 12, 10)

= -4.449

In Rupees (-4.449\*45) = Rs. -200.18

C. Since the standard deviation of the second division is 4 and is more than the standard deviation of the first division which is 3, the second division has a larger probability of making a loss in a given year.